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The β Function, Scaling, and Improved Action for $SU(3)$ Lattice Gauge Theory

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We present a detailed analysis of the nonperturbative β function along the Wilson axis for the $SU(3)$ pure gauge theory using the Monte Carlo renormalization group method. The scaling behavior of the string tension, the deconfinement transition temperature, and the O^{++} glueball mass obtained from published data is compared. The results show that there is no asymptotic scaling for $K_F \equiv (6/g^2) < 6.1$. We also estimate the renormalized action generated by the $\sqrt{3}$ block transformation for use in future calculations.

KEY WORDS: β function; lattice gauge theory, MCRG, improved action, $SU(3)$.

To extract the continuum properties of the theory, one needs to know how a particle mass, measured in units of the lattice spacing a, changes with the coupling g_{bare}, i.e., how a physical quantity scales. In the asymptotic region region (near $g_{bare} \sim 0$). this scaling is given by the 2-loop perturbative β function

$$\frac{\partial(g^{-2})}{\partial(\ln a)} = -\frac{1}{8\pi^2} - \frac{51}{64\pi^4} g^2 + \cdots \tag{1}$$

The quantity we calculate using MCRG is[1]

$$\Delta K_F = -\frac{\Delta(6g^{-2})}{\Delta(\ln a)} \cdot \ln \sqrt{3} \tag{2}$$

i.e., the discrete β function at K_F evaluated for a scale change of $\sqrt{3}$. The same quantity can be extracted from a physical observable m as follows:

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Let $ma_1(ma_2)$ be the measured values at coupling $K_F^1(K_F^2)$. Then the ΔK_F, scaled to correspond to a scale change b for comparison, is given by

$$\Delta K_F = (K_F^1 - K_F^2) \frac{\ln(b)}{\ln(ma_2/ma_1)} \tag{3}$$

The results are shown in Fig. 1.

In comparing the $b = \sqrt{3}$ MCRG results with the "β functions" obtained from observables like the string tension,[2] the deconfining transition temperature,[3] and the O^{++} glueball mass,[4] one encounters the problem of adjusting scales. The operational solution we have adopted is to only choose pairs of data points with a scale change within 20% of $\sqrt{3}$, rescale the ΔK_F to $b = \sqrt{3}$, and then plot the scaled ΔK_F at the midpoint of the original interval. We find $b = \sqrt{3}$ large enough so that the errors in the

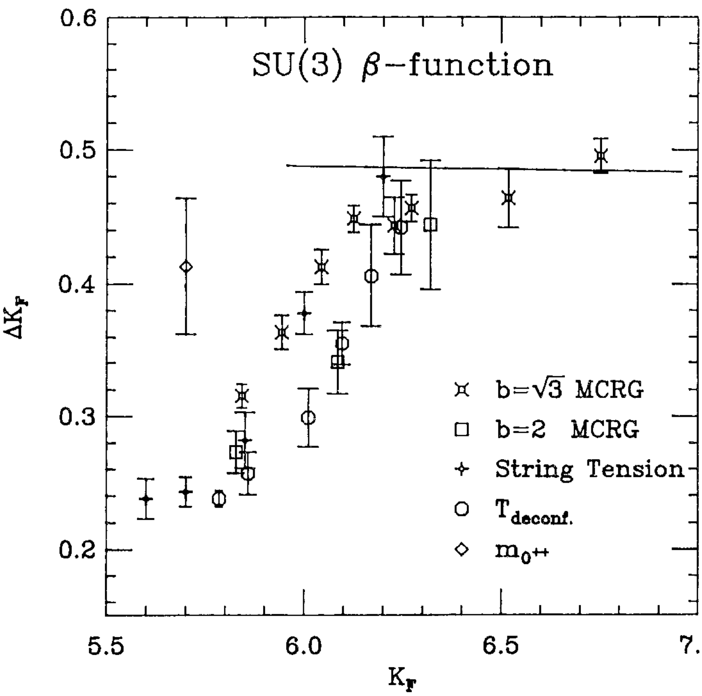

Fig. 1. Comparison of the $b = \sqrt{3}$ MCRG ΔK_F with the 2-loop perturbative result (solid line). Also shown are the ΔK_F evaluated from the $b = 2$ MCRG, string tension, deconfinement transition temperature, and the O^{++} glueball mass data.

individual measurements are small compared to the differences. However, to detect fine structure in the β function a still smaller value of b might be necessary.

The $b=2$ MCRG data[5] falls slightly below our results. This difference could be due to the rescaling of the ΔK_F. In calculating the ΔK_F by MCRG, we have effectively held the string tension constant, i.e., matched block Wilson loops. The three sets of data for σa^{2}[2] have about 20% spread in their lattice values. We therefore calculated ΔK_F separately for each set, and these results were found to be more consistent with each other. We regard the spread as a realistic measure of the systematic errors still present in the calculations. As shown in Fig. 1, the $b=\sqrt{3}\,\Delta K_F$ do agree within errors with the string tension results. The ΔK_F from the deconfining transition temperature data show a dip extending to weaker couplings and are in better agreement with the $b=2$ MCRG results. The ΔK_F from the O^{++} glueball mass data is in disagreement with all the rest and does not show the same pronounced dip. We hope that the situation will be clarified as better data becomes available at weaker couplings for all these long distance observables.

The $SU(3)$ action in the $[K_F, K_6, K_8, K_{6p}]$ space is defined to be

$$S=\mathrm{Re}\left\{K_F\sum \operatorname{tr} U_p + K_{6p}\sum \operatorname{tr} U_{6p} + K_6 \sum \left[\tfrac{3}{2}(\operatorname{tr} U_p)^2 - \tfrac{1}{2}\operatorname{tr} U_p\right]\right\} + K_8\sum\left[\tfrac{9}{8}|\operatorname{tr} U_p|^2 - \tfrac{1}{8}\right] \tag{4}$$

Here the higher representations have been constructed from U_p, all the traces are normalized to unity, and the sums are over all sites and positive orientations of the loops. Our first-step estimate for a single parameter improved action is,[6]

$$\frac{K_6}{K_F}=\frac{K_8}{K_F}=-0.12, \qquad \frac{K_{6p}}{K_F}=-0.04 \tag{5}$$

The feature to take note of in (5) is that the contributions of the higher representations and the 6-link loops are not small. We have undertaken a detailed calculation of QCD mass-ratios using the action defined by Eq. (5) to check for improved scaling.

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